Int. J. Heat Mass Transfer. Vol. 12, pp. 129–136. Pergamon Press 1969. Printed in Great Britain

PIONEER PAPER

INTRODUCTION

THIS journal has, in the past, reprinted various papers which had a profound influence on our understanding of heat- and mass-transfer processes. The first paper in this series was one by Osborne Reynolds: On the Extent and Action of the Heating Surface of Steam Boilers (*Scientific Papers of Osborne Reynolds*, Vol. I, pp. 81–85. Cambridge University Press, London, 1901) reprinted in Vol. 3, No. 2 of this journal. The paper presents considerations which lead to the establishement of the analogy between convective heat and momentum transfer usually referred to as Reynolds analogy. Just one hundred years ago, in 1868, Osborne Reynolds was appointed to a chair at the University of Manchester. To commemorate this event, we are publishing a second paper: On the Two Manners of Motion of Water (*Proceedings of the Royal Institution of Great Britain*, 1884). This contribution summarizes Reynolds studies on the transition from laminar to turbulent flow which had been published in detail in the paper An Experimental Investigation of the Circumstances Which Determine Whether the Motion of Water Shall Be Direct or Sinuous, and of the Law of Resistance in Parallel Channels (*The Philosphical Transactions of the Royal Society*, 1883). A knowledge of the conditions for transition is a prerequisite to heat transfer analysis. It is revealing what insight into the factors causing transition Reynolds already had.

E. R. G. ECKERT

University of Minnesota
Minneapolis
Minnesota 55455

PAPERS ON MECHANICAL AND PHYSICAL SUBJECTS

OSBORNE REYNOLDS, F.R.S., Mem. Inst. C.E., L.L.D.
Professor of Engineering in the Owens College, and Honorary Fellow of Queens' College, Cambridge

Reprinted from Various Transactions and Journals.
Volume II, 1881–1900.
Cambridge, at the University Press (1901)

48

ON THE TWO MANNERS OF MOTION OF WATER

(From the *Proceedings of the Royal Institution of Great Britain*, 1884)

(*Read March* 28, 1884)

INTRODUCTION

IT HAS long been a matter of very general regret with those who are interested in natural philosophy, that in spite of the most strenuous efforts of the ablest mathematicians, the theory of fluid motion fits very ill with the actual behaviour of fluids; and this for unexplained reasons. The theory itself appears to be very tolerably complete, and affords the means of calculating the results to be expected in almost every case of fluid motion, but while in many cases the theoretical results agree with those actually obtained, in other cases they are altogether different.

If we take a small body such as a raindrop moving through the air, the theory gives us the true law of resistance; but if we take a large body such as a ship moving through the water, the theoretical law of resistance is altogether out. And what is the most unsatisfactory part of the matter is that the theory affords no clue to the reason why it should apply to the one class more than the other.

When, seven years ago, I had the honour of lecturing in this room on the then novel subject of vortex motion, I ventured to insist that the reason why such ill success had attended our theoretical efforts was because, owing to the uniform clearness or opacity of water and air, we can see nothing of the internal motion; and while exhibiting the phenomena of vortex rings in water, rendered strikingly apparent by partially colouring the water, but otherwise as strikingly invisible, I ventured to predict that the more general application of this method, which I may call the method of colour-bands, would reveal clues to those mysteries of fluid motion which had baffled philosophy.

To-night I venture to claim what is at all events a partial verification of that prediction. The fact that we can see as far into fluids as into solids naturally raises the question why the same success should not have been obtained in the case of the theory of fluids as in that of solids? The answer is plain enough. As a rule, there is no internal motion in solid bodies; and hence our theory based on the assumption of relative internal rest applies to all cases. It is not, however, impossible that an, at all events seemingly, solid body should have internal motion, and a simple experiment will show that if a class of such bodies existed they would apparently have disobeyed the laws of motion.

These two wooden cubes are apparently just alike, each has a string tied to it. Now, if a ball is suspended by a string you all know that it hangs vertically below the point of suspension or swings like a pendulum. You see this one does so. The other you see behaves quite differently, turning up sideways. The effect is very striking so long as you do not know the cause. There is a heavy revolving wheel inside which makes it behave like a top.

Now what I wish you to see is, that had such bodies been a work of nature so that we could not see what was going on—if, for instance, apples were of this nature while pears were what they are—the laws of motion would not have been discovered; if discovered for pears they would not have applied to apples, and so would hardly have been thought satisfactory.

Such is the case with fluids: here are two vessels of water which appear exactly similar—even more so than the solids, because you can see right through them—and there is nothing unreasonable in supposing that the same laws of motion would apply to both vessels. The application of the method of colour-bands, however, reveals a secret: the water of the one is at rest, while that in the other is in a high state of agitation.

I am speaking of the two manners of motion of water—not because there are only two motions possible; looked at by their general appearance the motions of water are infinite in number; but what it is my object to make clear to-night is that all the various phenomena of moving water may be divided into two broadly distinct classes, not according to what with uniform fluids are their apparent motions, but according to the internal motions of the fluids, which are invisible with clear fluids, but which become visible with colour-bands.

The phenomena to be shown will, I hope, have some interest in themselves, but their intrinsic interest is as nothing compared to their philosophical interest. On this, however, I can but slightly touch.

I have already pointed out that the problems of fluid-motion may be divided into two classes: those in which the theoretical results agree with the experimental, and those in which they are altogether different. Now what makes the recognition of the two manners of internal motion of fluids so important, is that all those problems to which the theory fits belong to the one class of internal motions.

The point before us to-night is simple enough, and may be well expressed by analogy. Most of us have more or less familiarity with the motion of troops, and we can well understand that there exists a science of military tactics which treats of the best manoeuvres and evolutions to meet particular circumstances.

Suppose this science proceeds on the assumption that the discipline of the troops is perfect, and hence takes no account of such moral effects as may be produced by the presence of an enemy.

Such a theory would stand in the same relation to the movements of troops, as that of hydrodynamics does to the movements of water. For although only the disciplined motion is recognised in military tactics, troops have another manner of motion when anything disturbs their order. And this is precisely how it is with water: it will move in a perfectly direct disciplined manner under some circumstances, while under others it becomes a mass of eddies and cross streams, which may be well likened to the motion of a whirling, struggling mob where each individual particle is obstructing the others.

Nor does the analogy end here: the circumstances which determine whether the motion of troops shall be a march or a scramble, are closely analogous to those which determine whether the motion of water shall be direct or sinuous.

In both cases there is a certain influence necessary for order: with troops it is discipline; with water it is viscosity or treacliness.

The better the discipline of the troops, or the more treacly the fluid, the less likely is steady motion to be disturbed under any circumstances. On the other hand, speed and size are in both cases influences conducive to unsteadiness. The larger the army, and the more rapid the evolutions, the greater the chance of disorder; so with fluid, the larger the channel, and the greater the velocity, the more chance of eddies.

With troops some evolutions are much more difficult to effect with steadiness than others, and some evolutions which would be perfectly safe on parade, would be sheer madness in the presence of an enemy. So it is with water.

One of my chief objects in introducing this analogy of the troops is to emphasise the fact, that even while executing manoeuvres in a steady manner, there may be a fundamental difference in the condition of the fluid. This is easily realised in the case of troops. Difficult and easy manoeuvres may be executed in equally steady manners if all goes well, but the conditions of the moving troops are essentially different. For while in the one case any slight disarrangement would be easily rectified, in the other it would inevitably lead to a scramble. The source of such a change in the manner of motion under such circumstances, may be ascribed either to the delicacy of the manoeuvre, or to the upsetting disturbance, but as a matter of fact, both of these causes are necessary. In the case of extreme delicacy an indefinitely small disturbance, such as is always to be counted on, will effect the change.

Under these circumstances we may well describe the condition of the troops in the simple manoeuvre as stable, while that in the delicate manoeuvre is unstable, i.e. will break down on the smallest disarrangement. The small disarrangement is the immediate source of the break-down in the same sense as the sound of a voice is sometimes the cause of an avalanche; but if we regard such disarrangement as certain to occur, then the source of the disturbance is a condition of instability.

All this is exactly true for the motion of water. Supposing no disarrangement, the water would move in the manner indicated in theory just as, if there is no disturbance, an egg will stand on its end; but as there is always slight disturbance, it is only when the condition of steady motion is more or less stable that it can exist. In addition then to the theories either of military tactics or of hydrodynamics, it is necessary to know under what circumstances the manoeuvres of which they treat are stable or unstable. And it is in definitely separating these conditions that the method of colour-bands has done good service which will remove the discredit in which the theory of hydrodynamics has been held.

In the first place, it has shown that the property of viscosity or treacliness, possessed more or less by all fluids, is the general influence conclusive to steadiness, while, on the other hand, space and velocity are the counter influence; and the effect of these influences is subject to one perfectly definite law, which is that a particular evolution becomes unstable for a definite value of the viscosity divided by the product of the velocity and space. This law explains a vast number of phenomena which have hitherto appeared paradoxical. One general conclusion is, that with sufficiently slow motion all manners of motion are stable.

The effect of viscosity is well shown by introducing a band of coloured water across a beaker filled with clear water at rest. Now the water is quite still, I turn the beaker round about its axis. The glass turns but not the water, except that which is close to the glass. The coloured water which is close to the glass is drawn out into what looks like a long smear, but it is not a smear, it is simply a colour-band extending from the point in which the colour touched the glass in a spiral manner inwards, showing that the viscosity was slowly communicating the motion of the glass to the water within. To prove this I have only to turn the beaker back, and the colour-band assumes its radial position. Throughout this evolution the motion has been quite steady—quite according to the theory.

When water flows steadily it flows in streams. Water flowing along a pipe is such a stream bounded by the solid surface of the pipe, but if the water be flowing steadily we can imagine the water to be divided by ideal tubes into a fagot of indefinitely small streams, any of which may be coloured without altering its motion, just as one column of infantry may be distinguished from another by colour.

If there is internal motion, it is clear that we cannot consider the whole stream bounded by the pipe as a fagot of elementary streams, as the water is continually crossing the pipe from one side to the other, any more than we can distinguish the streaks of colour in a human stream in the corridor of a theatre.

Solid walls are not necessary to form a stream: the jet from a fire hose, the falls of Niagara, are streams bounded by a free surface.

A river is a stream half bounded by a solid surface.

Streams may be parallel, as in a pipe; converging, as in a conical mouthpiece; or when the motion is reversed, diverging. Moreover, the streams may be straight or curved.

All these circumstances have their influence on stability in a manner which is indicated in the accompanying table:—

Circumstances conducive to

Direct or Steady Motion	*Sinuous or Unsteady Motion*
1. Viscosity or fluid friction which continually destroys disturbances. (Treacle is steadier then water.)	5. Particular variation of velocity across the stream, as when a stream flows through still water.
2. A free surface.	6. Solid bounding walls.
3. Converging solid boundaries.	7. Diverging solid boundaries.
4. Curvature with the velocity greatest on the outside.	8. Curvature with the velocity greatest on the inside.

It has for a long time been noticed that a stream of fluid through fluid otherwise at rest is in an unstable condition. It is this instability which gives rise to the talking-flame and sensitive-jet with which you have been long familiar in this room. I have here a glass vessel of clear water in front of the lantern, so that any colour-bands will be projected on the screen.

You see the ends of two vertical tubes one above the other. Nothing is flowing through these tubes, and the water in the vessel is at rest. I now open two taps, so as to allow a steady stream of coloured water to enter at the lower pipe, water flowing out at the upper. The water enters quite steadily, forms a sort of vortex ring at the end which proceeds across the vessel, and passes out at the lower tube. Now the coloured stream extends straight across the vessel, and fills both pipes. You see no motion; it looks like a glass rod. The water is, however, flowing slowly along it. The motion is so slow, that the viscosity is paramount, and hence the stream is steady.

I increase the speed; you see a certain wriggling sinuous action in the column; faster, the column breaks up into beautiful and well-defined eddies, and spreads out into the surrounding water, which, becoming opaque with colour, gradually draws a veil over the experiment.

The same is true of all streams bounded by standing water. If the motion is sufficiently slow, according to the size of the stream and the viscosity of the fluid, it is steady and stable. At a certain critical velocity, which is determined by the ratio of the viscosity to the diameter of the stream, the stream becomes unstable. Under any conditions, then, which involve a stream flowing through surrounding water, the motion will be unstable if the velocity is sufficient.

Now, *one* of the most marked facts relating to experimental hydrodynamics is the difference in the way in which water flows along contracting and expanding channels; these include an enormously large class of the motions of water, but a typical phenomenon is shown by the simple conical tubes. Such a tube is now projected on the screen; it is surrounded with clear still water. The mouth of the tube at which the water enters is the largest part, and it contracts uniformly for some way down the channel, then the tube expands again gradually until it is nearly as large as at the mouth, and then again contracts to the tube necessary to discharge the water. I draw water through the tube, but you see nothing as to what is going on. I now colour one of the elementary streams outside the mouth; this colour-band is drawn in with the surrounding water, and will show us what is going on. It enters quite steadily, preserving its clear streak-like character until is has reached the neck where convergence ceases; now the moment it enters the expanding tube it is altogether broken up into eddies. Thus the motion is direct in the contracting tube, sinuous in the expanding.

The hydrodynamical theory affords no clue to the cause why; and even by the method of colour-bands the reason for the sinuosity is not at once obvious. If we start the current suddenly, the motion is at first the same in both tubes, its change in the expanding pipe seemed to imply that here the motion was unstable. If so, this ought to appear from the equations of motion. With this view this case was studied, I am ashamed to say how long, without any light. I then had recourse to the colour-bands again, to try and see how the phenomena came on. It all then became clear: there is an intermediate stage. When the tap is opened, the immediately ensuing motion is nearly the same in both parts; but while that in the contracting portion maintains its character, that in the expanding portion changes its character. A vortex ring is formed which, moving forward, leaves the motion behind that of a parallel stream through the surrounding water.

If the motion be sufficiently slow, as it is now, this stream is stable, as already explained. We thus have steady or direct motion in both the contracting and expanding parts of the tube, but the two motions are not similar: the first being one of a fagot of similar elementary contracting streams, the latter being that of one parallel stream through the surrounding fluid. The first of these is a stable form; the second an unstable form, and, on increasing the velocity, the first remains, while the second breaks down; and we have, as before, the expanding part filled with eddies.

This experiment is typical of a large class of motions. Wherever fluid flows through a narrow, as it approaches the neck it is steady, after passing, it is sinuous. The same effect is produced by an obstacle in the middle of a stream; and very nearly the same thing by the motion of a solid object through the water.

You see projected on the screen an object not unlike a ship. Here the ship is fixed, and the water flowing past it; but the effect would be the same if we had the ship moving through the water. In the front of the ship the stream is steady, and so till it has passed the middle, then you see the eddies formed behind the ship. It is these eddies which account for the discrepancy between the actual and theoretical resistance of ships. We see, then, that the motion in the expanding channel is sinuous because the only steady motion is that of a stream through water. Numerous cases in which the motion is sinuous may be explained in the same way, but not all.

If we have a perfectly parallel channel, neither contracting nor expanding, the steady moving stream will be a fagot of perfectly steady parallel elementary streams all in motion, but moving fastest at the centre. Here we have no stream through steady water. Now when this investigation began it was not known, or imperfectly known, whether such a stream was stable or not, but there was a well-known anomaly in the resistance to motion in parallel channels. In rivers, and all pipes of sensible size, experience had shown that the resistance increased as the square of the velocity, whereas in very small pipes, such as represent the smaller veins in animals, Poiseuille had proved the resistance increased as the velocity.

Now since the resistance would be as the square of the velocity with sinuous motion, and as the velocity, if direct, it seemed that the discrepancy could be accounted for if the motion could be shown to become unstable for a sufficiently large velocity. This suggested the experiment I am now about to produce before you.

You see on the screen a pipe with its end open. It is surrounded by clear water, and by opening a tap I can draw water through it. This makes no difference to the appearance, until I colour one of the elementary streams, when you see a beautiful streak of colour extend all along the pipe. The stream has so far been running steadily, and appears quite stable. I now merely increase the speed; it is still steady, but the colour-band is drawn down fine. I increase the colour and then again increase the speed. Now you see the colour-band at first vibrates and then mixes so as to fill the tube. This is at a definite velocity; if the velocity be diminished even so little the band becomes straight and clear; increase it again, it breaks up. This critical speed depends on the size of the tube in the exact inverse ratio; the smaller the tube, the greater the velocity; also, the more viscous the water the greater the velocity.

We have then not only a complete explanation of the difference in the laws of resistance generally experienced, and that found by Poiseuille, but also we have complete evidence of the instability of parallel streams flowing between or over solid surfaces. The cause of the instability is as yet not explained, but this much can be shown, that whereas lateral stiffness in the walls is unimportant, inextensibility or tangential rigidity is essential to the creation of eddies. I cannot show you this, because the only way in which we can produce the necessary conditions without a solid channel, is by a wind blowing over water. When the wind blows over water, it imparts motion to the surface of the water just as a moving solid surface; moving in this way, however, the water is not susceptible of eddies. It is unstable, but the result of disturbance is waves. This is proved by an experiment long known, but which has recently attracted considerable notice. If oil be put on the surface it spreads out into an indefinitely thin sheet which possesses only one of the characteristics of a solid surface, it offers resistance, very slight, but still resistance to extension and contraction. This, however, is sufficient to entirely alter the character of the motion. It renders the water unstable internally, and instead of waves, what the wind does is to produce eddies beneath the surface. This has been proved, although I cannot show you the experiments.

To those who have observed the phenomena of oil preventing waves, there is probably nothing more striking throughout the region of mechanics. A film of oil so thin that we have no means of illustrating its thickness, and which cannot be perceived except by its effect—which possesses no mechanical properties that can be made apparent to our senses—is yet able to entirely prevent an action which involves forces the strongest we can conceive, which upset our ships and destroy our coasts. This, however, becomes intelligible when we perceive that the action of the oil is not to calm the sea by sheer force, but merely, as by its moral force, to alter the manner of motion produced by the action of the wind, from that of the terrible waves upon the surface, into the harmless eddies below. The wind throws the water into a highly unstable condition, into what morally we should call a condition of great excitement. The oil by an influence we cannot perceive directs this excitement.

This influence, though insensibly small, is however now proved of a mechanical kind, and to me it seems that the phenomenon of one of the most powerful mechanical actions of which the forces of nature are capable, being entirely controlled by a mechanical force so slight as to be otherwise quite imperceptible, does away with every argument against the strictly mechanical sources of what we may call mental and moral forces.

But to return to the instability in parallel channels. This has been the most complete, as well as the most definite result of the colour-bands.

The circumstances are such as to render definite experiments possible. These have been made, and reveal a definite law of the instability, which law has been tested by reference to all the numerous and important experiments on the resistance in channels by previous observers; whereupon it is found that waters behave in exactly the same manner whether the channel, as in Poiseuille's experiment, is of the dimensions of a hair, or whether it be the size of a water main or of the Mississippi; the only difference being that in order that the motions may be compared, the velocity must be inversely as the diameter of the pipe. But this is not the only point explained if we consider other fluids than water. Some fluids, like oil or treacle, apparently flow more slowly and steadily than water. This, however, is only in smaller channels; the critical velocity increases with the viscosity of the fluid. Thus, while water in comparatively large streams is always above its critical velocity, and the motion always sinuous, the motion of treacle in streams of such size as we see is below its critical velocity, and the motion direct. But if nature had produced rivers of treacle the size of the Thames, for instance, the treacle would have flowed just like water. Thus, in the lava streams from a volcano, although looked at close the lava has the consistence of a pudding, in the large and rapid streams down the mountain sides the lava flows as freely as water.

I have now only one circumstance left to which to ask your attention. This is the effect of the curvature of the stream on the stability of the fluid.

Here again we see the whole effect altered by very slight causes.

If water be flowing in a bent channel in steady streams, the question as to whether it will be stable or not turns on the variation in the velocity from the inside to the outside of the stream.

In front of the lantern is a cylinder with glass ends, so that the light passes through in the direction of the axis. The disk of light on the screen being the light which passes through this water, and is bounded by the circular walls of the cylinder.

By means of two tubes temporarily attached, a stream of coloured water is introduced right across the cylinder extending from wall to wall; the motion is very slow, and the taps being closed, and the tubes removed, the colour-band is practically stationary. The vessel is now caused to revolve about its axis. At first, only the walls of the cylinder move, but the colour-band shows that the water gradually takes up the motion, the streak being wound off at the ends into a spiral thread, but otherwise remaining still and vertical. When the spirals meet in the middle, the whole water is in motion, but the motion is greatest at the outside, and is therefore stable. The vessel stops, and gradually stops the water, beginning at the outside. If the motion remained steady, the spirals would unwind, and the streak be restored. But the motion being slowest at the outside against the surface, you see eddies form, breaking up the spirals for a certain distance towards the middle, but leaving the middle revolving steadily.

Besides indicating the effect of curvature, this experiment really illustrates the action of the surface of the earth on the air moving over it; the varying temperature having much the same influence as the curvature of the vessel on stability. The air is unstable for a few thousand feet above the surface, and the motion is sinuous, resulting in the mixing of the strata, and producing the heavy cumulus clouds; but above this the influence of temperature predominates, and clouds, if there are any, are of the stratus-form, like the inner spirals of colour. But it was not the intention of this to trace the two manners of motion of fluids in the phenomena of Nature and Art, so I thank you for your attention.